Tiny House Calculation Packet

ENGS 71 STRUCTURAL ANALYSIS

Trellis Design

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1 Design Philosophy

The Tiny House is being designed for use by the Dartmouth Organic Farm as a meeting space. The house is designed to stand on a slope facing the river, and has a large deck where staff and visitors can see the farm and the river. The house is being build with sustainability practices reflective of those practiced by the organic farm, with emphasis placed on using locally produced materials and using building techniques that offer minimal environmental impact.

The trellis sits eight feet above the river-facing portion of the deck. The chosen design uses four posts to support the dead and snow loads. While a combination of posts and cables is a more attractive option, the cost of materials is much higher than an all-post design. Both options are presented such that the designers can choose whichever option meets the cost constraints of the building. All calculations are based on data from National Design Specification (NDS) for wood and International Building Code (IBC).

2 Site

Figure 1: Site Plan for Tiny House

The architecture class chose a site sitting on the steep slope that faces the river. The site offers views of the river and farm below while still being easily accessible from the farm's parking area.

3 Design Concept

The following is a potential design for the trellis of the tiny house. This report details the design of a 4-post supported trellis, but also offers information on design for a 2-post, 2-cable solution.

Figure 2: Floor Plan for Tiny House and Trellis

The trellis will span 27', from locations 2 to 5 in the diagram above. Supports are necessary to hold up a beam on line A. This can include posts connected to the deck on line A or cables, connected to the roof at the intersection of lines B and 5 and extending to various points along the beam on line A.

4 Loads

The loads in consideration for the trellis design are the dead and snow loads.

Dead Load

The dead load is 15 pounds per square foot.

Snow Load

The snow load in Hanover, New Hampshire is 60 pounds per square foot.

Combined Load

the load was calculated to be 50% of the combined dead and snow load. This is because the width of the trellis slats and their spacing are both 4", so 50% of the area covered by the trellis is open and will allow snow to fall through. The tributary width for the beam on line A is $7'/2$.

$$
0.5 * (D + S) * W_T = 0.5 * (15 + 60) * 7/2 = 131.25 \text{lb/ft}
$$
 (1)

5 Trellis Design

We first calculate the appropriate size for the slats that make up the trellis, which will be used for all subsequent support design options.

We then calculate the necessary post, cable, and beam sizes for the given load with multiple design options: 1 post, 2 posts, multiple posts, 1 cable, 2 cables, and a combination of posts and cables. All calculations are done using Northern White Cedar's properties.

Following is the summary of results from all computations. While most calculations offer several different sizes for beams and posts, the displayed size is that which is deemed most cost effective based on research of lumber prices from Fogg's, Baker Lumber, Lowe's and Menard's, wire prices from McMaster-Carr, and clevis prices from Hanes Supply. Note that the costs may be an underestimate, as very few prices could be found for the beam that spans 27'.

The green row is the recommended design option. The yellow row is the secondary design option, should the builders desire cables as supports despite the higher cost.

5.1 Table of Results

Table 1: Design Options

5.2 Trellis Slat Size

For the given load of $75 * 0.5$ psf, with an initial size of $3"x2"$, the distributed load is found to be 9.375 plf. This load spans a 7' beam, for a total load of 65.625 lb.

$$
\sum M_A = 65.625(7/2) - 7B_y = 0 \rightarrow B_y = 32.8125 \,\text{lb} \tag{2}
$$

$$
B_y = A_y = 32.8125 \,\text{lb} \tag{3}
$$

$$
M_{max} = 31.825 \times 7/2 = 114.844 \,\text{lb} \times \text{ft} \tag{4}
$$

$$
\Delta_{max} = \frac{5wl^4}{384EI} = \frac{5(9.375)(7*12)^4}{384(700000)(1.953)} = 4.446 \,\text{in} \tag{5}
$$

$$
\Delta_{allow} = L/120 = 0.7 \,\text{in} \tag{6}
$$

Note that $\Delta_{max} > \Delta_{allow},$ so this size is not allowable.

Next we try a 4"x4" slat. Here the distributed load is 12.5 plf, and the total load is 87.5 lb.

$$
\sum M_A = 87.5(7/2) - 7B_y = 0 \rightarrow B_y = 43.75 \,\text{lb} \tag{7}
$$

$$
B_y = A_y = 43.75 \,\text{lb} \tag{8}
$$

$$
M_{max} = 43.75 * 7/2 = 153.125 \,\text{lb} * \text{ft} \tag{9}
$$

$$
\Delta_{max} = \frac{5wl^4}{384EI} = \frac{5(12.5)(7*12)^4}{384(700000)(21.33)} = 0.543 \,\text{in} \tag{10}
$$

$$
\Delta_{allow} = L/120 = 0.7in \tag{11}
$$

Note that $\Delta_{max} < \Delta_{allow},$ so this size is acceptable. The slats for all calculations and designs moving forward will therefore be 4"x4".

5.3 1 Post Design

First, the reactions at the supports must be calculated.

$$
\sum M_2 = 131.25 \times 27 \times 13.5 - 5_y \times 27 = 0 \tag{12}
$$

$$
\sum F_y = -131.25 \times 27 + 2_y + 5_y = 0 \tag{13}
$$

$$
5_y = 2_y = 1771.875 \,\text{lb} \tag{14}
$$

Figure 4: Shear and Moment Diagrams

The maximum moment for the beam can be used to find the minimum section modulus. \overline{M}

$$
\frac{M}{S} = f_b \le F_b * C_D * C_r * C_F \tag{15}
$$

$$
\frac{11960 = 143521.875 \,\text{lb} \,\ast \,\text{in}}{S} \le 550 \,\ast 1.15 \,\ast 1.15 \,\ast 1 \tag{16}
$$

Note that the adjustment factors $C_F = 1$ is temporary. Once an initial beam size is found, the section modulus will be re-calculated.

$$
S_{min} = 197.315 \,\mathrm{in}^3 \tag{17}
$$

This section modulus requires timber of size 10x12. This size is unreasonable for the trellis, so design options with more supports will be considered.

5.4 2 Post Design

Figure 5: Representation of 2 post design option

First, the reactions at the supports must be calculated. This beam is 1°indeterminate, so the Force Method will be employed to find all reactions.

The determinate beam is modeled as a pin-roller beam, with supports on either end of the beam.

Deflection at $L/2$:

$$
\frac{-5wL^4}{384EI} = \frac{-5(131.25)(27)^4}{284EI} = \frac{-908224.3625}{EI}
$$
(18)

Apply the redundant at $L/2$:

$$
\frac{PL^3}{48EI} = \frac{\frac{L}{2}y(27)^3}{48EI} = \frac{\frac{L}{2}y * 410.0625}{EI}
$$
(19)

$$
\frac{L}{2}y = 2214.84375 \,\text{lb} \tag{20}
$$

$$
\sum M_2 = 131.25 \times 27(13.5) - 2214.84(13.5) - 5_y(27) = 0 \tag{21}
$$

$$
5_y = 664.45 \,\mathrm{lb} \tag{22}
$$

$$
\sum F_y = 2_y + \frac{L}{2_y} + 5_y - 131.25 \times 27 \tag{23}
$$

$$
2_y = 664.45 \,\mathrm{lb} \tag{24}
$$

Figure 6: Shear and Moment Diagrams

The maximum moment for the beam can be used to find the minimum section modulus. $\ddot{}$

$$
\frac{M}{S} = f_b \le F_b * C_D * C_r * C_F \tag{25}
$$

$$
\frac{2990.3 = 25883.6 \,\text{lb} \ast \text{in}}{S} \le 550 \ast 1.15 \ast 1.15 \ast 1 \tag{26}
$$

Note that the adjustment factors $C_F = 1$ is temporary. Once an initial beam size is found, the section modulus will be re-calculated.

$$
S_{min} = 49.333 \,\mathrm{in}^3 \tag{27}
$$

This Section modulus corresponds to the following lumber sizes: 3x12, 4x10. The revised C_F is 1.15.

$$
\frac{2990.3 = 25883.6 \,\mathrm{lb} \,\mathrm{*} \,\mathrm{in}}{S} \le 550 \,\mathrm{*} \,1.15 \,\mathrm{*} \,1.15 \,\mathrm{*} \,1 \,\mathrm{*} \,1.15 \tag{28}
$$

$$
S_{min} = 42.898 \,\mathrm{in}^3 \tag{29}
$$

The two previous sizes still satisfy the requirement for the section modulus. Another lumber size that satisfies the requirement is 2x14.

For all applicable sizes, check that $\Delta_{max} < \Delta_{allow}$. For 3x12:

$$
\Delta_{max} = \frac{5wl^4}{384EI} = 0.472 \,\text{in} < \Delta_{allow} = \frac{L}{120} = 1.35 \,\text{in}
$$
\n(30)

For 4x10:

$$
\Delta_{max} = \frac{5wl^4}{384EI} = 0.607 \text{ in} < \Delta_{allow} = \frac{L}{120} = 1.35 \text{ in}
$$
 (31)

For 2x14:

$$
\Delta_{max} = \frac{5wl^4}{384EI} = 0.48 \text{ in} < \Delta_{allow} = \frac{L}{120} = 1.35 \text{ in}
$$
 (32)

All sizes have $\Delta_{max} < \Delta_{allow}$.

 2°

Check all sizes for shear failure. v_{max} must be less than v_{allow} .

$$
f_v = \frac{VQ}{Ib} = \frac{3v}{2bd} \tag{33}
$$

$$
F_v* = F_b*C_D*C_r = 550*1.15*1.15
$$
\n(34)

For 3x12:

$$
\frac{3v}{2(3)(12)} = 727.375 \rightarrow v_{allow} = 17457
$$
 (35)

$$
v_{max} = 1107.42 \rightarrow v_{max} < v_{allow} \tag{36}
$$

For 4x10:

$$
\frac{\partial v}{2(4)(10)} = 727.375 \rightarrow v_{allow} = 19396.667\tag{37}
$$

$$
v_{max} = 1107.42 \rightarrow v_{max} < v_{allow} \tag{38}
$$

For 2x14:

$$
\frac{3v}{2(2)(14)} = 727.375 \rightarrow v_{allow} = 40733
$$
 (39)

$$
v_{max} = 1107.42 \rightarrow v_{max} < v_{allow} \tag{40}
$$

Note that there are no deflection or shear failures in the applicable beam sizes.

Next, the columns must be designed, using the maximum shear value of 1107.42 lb. The initial assumptions are that $C_F = 1$ and the dimension is 4x.

$$
\frac{\text{Force}}{\text{Area}} = \sigma_{axial} = F_c \le F_c * C_p * C_D * C_F \tag{41}
$$

$$
\frac{1107.42}{A} \le 475 \times C_p \times 1.15 \times 1\tag{42}
$$

$$
F_c* = 546.25\tag{43}
$$

$$
C_p = \frac{1 + (F_{CE}/F_{C}*)}{2c} - \sqrt{\left[\frac{1 + (F_{CE}/F_{C}*)}{2c}\right]^2 - \frac{F_{CE}/F_{C}*}{c}}
$$
(44)

$$
F_{CE} = \frac{K_{CE} E'}{(c/d)^2} \tag{45}
$$

Note that $c = 0.8$ for sawn lumber, and $K_{CE} = 0.822$.

$$
C_p = 0.547\tag{46}
$$

$$
\frac{1107.42}{A} \le 475 \times 0.547 \times 1.15 \times 1 \tag{47}
$$

$$
A = 3.7058 \,\mathrm{in}^2 \tag{48}
$$

This area allows for a $4x4$ post $(A = 12.25 \text{ in}^2)$. Revise C_p assuming 3x dimension

$$
C_p = 0.3456 \tag{49}
$$

$$
A = 5.866 \,\mathrm{in}^2 \tag{50}
$$

This area allows for a 3x6 post $(A = 12.60 \text{ in}^2)$. Revise C_p assuming 2x dimension

$$
C_p = 0.1634 \tag{51}
$$

$$
A = 12.407 \,\mathrm{in}^2 \tag{52}
$$

This area allows for a $2x10$ post $(A = 13.88 \text{ in}^2)$

5.5 1 Cable Design

The cable will be mounted to the tallest point on the roof. A roof slope of 4:12 is expected.

Figure 7: 1 Cable Design

$$
\sum M_2 = 13.5(131.25 \times 27) - 27 \times 5_y = 0 \rightarrow 5_y = 1771.875 \,\text{lb} \tag{53}
$$

$$
\sum F_y = 2_y + 5_y - 131.25 \times 27 \to 2_y = 1771.875 \,\text{lb} \tag{54}
$$

$$
M_{max} = \frac{wl^2}{8} = \frac{131.25 \times 27^2}{8} = 11960 \,\text{lb} \times \text{ft} \tag{55}
$$

$$
\frac{M}{S} \le F'_B = F_B * C_D * C_F * C_r \tag{56}
$$

$$
\frac{11960*12}{S} \le 550*1.15*1*1.15
$$
 (57)

$$
S \ge 197.3 \,\mathrm{in}^3 \tag{58}
$$

This section modulus requires a beam of size 10 x 12, which is unreasonably large. Therefore, designs with more than two support points must be considered.

5.6 2 Cable Design

The cables are placed at locations 2 and $L/2$, and are modeled to be perpendicular to the beam and connect to the edge of the roof. For a 3:12 roof slope, the height of the cable at location 2, the very end of the beam, must be 1'. For a 4:12 roof slope, the height must be 1.33'. For the cable at $L/2$, the cable height must be 4.375' for the 3:12 roof slope and 5.83' for the 4:12 roof slope.

For cable 1 at location 2 with a 3:12 roof slope, the length of the cable is 7.07'. For a 4:12 slope, the length is 7.13'.

For cable 2 at $L/2$ with a 3:12 roof slope, the length of the cable is 8.25'. For a 4:12 slope, the length is 9.11'.

Figure 8: 2 Cable Design

The design is 2°indeterminate. The slope-deflection method is used to find the reaction forces. The degrees of freedom, with axial deformations ignored, are Θ_{C1} and Θ_{C2} , where C1 refers to cable 1 at location 2, and C2 refers to cable 2 at $L/2$.

Figure 9: 2 cable slope-deflection regular load

Figure 10: 2 cable slope-deflection Θ_{C2}

6El*Theta C1 / 182.25

Figure 11: 2 cable slope-deflection Θ_{C1}

$$
\sum M_{C1} = \frac{4EI\Theta_{C1}}{13.5} + \frac{2EI\Theta_{C2}}{13.5} + 1993.36 = 0
$$
 (59)

$$
\sum M_{C2} = \frac{2EI\Theta_{C1}}{13.5} + \frac{4EI\Theta_{C2}}{13.5} + \frac{4EI\Theta_{C2}}{13.5} = 0
$$
 (60)

Solving these equations,

$$
EI\Theta_{C2} = 1922.17, EI\Theta_{C1} = -7688.67\tag{61}
$$

The reactions are found to be

$$
F_{yC1} = 696.1 \,\text{lb}, \, F_{yC2} = 2025 \,\text{lb}, \, F_{y5} = 822.75 \,\text{lb} \tag{62}
$$

For cable 1, with a roof slope of 3:12, $F_{C1} = 9842.733$ lb. For a slope of 4:12, F_{C1} = 7463.395 lb. McMaster-Carr does not sell any cables that are properly rated for a 10000 lb force. However, a 5/8" diameter wire rope rated for 8200 lb would work for a 4:12 slope. Note that this design option cannot be easily used if the roof slope is 3:12.

For cable 2, with a roof slope of 3:12, $F_{C2} = 7637.14$ lb. For a slope of 4:12, $F_{C2}=6325.30$ lb. For the 3:12 case, a $5/8"$ diameter wire rope would be used. For 4:12, a 9/16" diameter rope could be used.

Figure 12: 2 Cable Shear and Moment Diagram

 $M_{max} = 2562.89$ at $L/2$. To find the beam size,

$$
\frac{2562.89 * 12}{S} \le F'_b = 550 * 1 * 1.15 * 1
$$
\n(63)

$$
S \ge 37.01 \,\mathrm{in}^3 \tag{64}
$$

This section modulus allows for beam sizes of 4x10 or 3x12. Revisiting the size factor C_F , which now equals 1.2 for the 4x10 beam, $S = 40.52$. However, this change in section modulus does not change the beam sizing selections.

For the post at the end of the beam, the shear is 822.75 lb. Using equation 41,

$$
\frac{822.75}{A} \le 475 \times C_p \times 1.15 \times 1\tag{65}
$$

Using equation 44, assuming a dimension of $4x$, C_P is found to equal 0.547.

$$
A \ge 2.75 \text{in}^2 \tag{66}
$$

Given this area, a 4x4 post could be used.

Assuming a dimension of 3x, C_P is found to equal 0.3456.

$$
A \ge 4.36 \text{in}^2 \tag{67}
$$

Given this area, a 3x4 post could be used.

Assuming a dimension of $2x$, C_P is found to equal 0.1634.

$$
A \ge 9.22 \text{in}^2 \tag{68}
$$

Given this area, a 2x8 post could be used.

5.7 1 Post 1 Cable Design

Figure 13: Representation of 1 post 1 cable design option

First, the reactions at the supports must be calculated. Since the design is 1 ◦ indeterminate, the Force Method will be used.

The statically determinate beam is a pin-roller beam with supports on either end of the beam.

$$
\delta_{max} = -\frac{5wl^4}{384EI} \tag{69}
$$

The redundant is an upwards force applied at $L/2$.

$$
\delta_{max} = \frac{\frac{L}{2} \, _y \, L^3}{48EI} \tag{70}
$$

Apply compatibility equation $\delta_{det}+\delta_{red}=0.$

$$
\frac{L}{2}y = 2214.84 \,\mathrm{lb} \tag{71}
$$

$$
\sum M_5 = -2214.84 * 13.5 + 131.25 * 27 * 13.5 - 2_y * 27 = 0 \tag{72}
$$

$$
\sum F_y = -131.25 \times 27 + 2_y + 5_y = 0 \tag{73}
$$

$$
5_y = 2_y = 664.455 \,\text{lb} \tag{74}
$$

Figure 14: Shear and Moment Diagrams

The maximum moment for the beam can be used to find the minimum section modulus. $\ddot{}$

$$
\frac{M}{S} = f_b \le F_b * C_D * C_r * C_F \tag{75}
$$

$$
\frac{2989.9 = 35878.8 \,\text{lb} \,\ast \,\text{in}}{S} \le 550 \,\ast \,1.15 \,\ast \,1.15 \,\ast \,1 \tag{76}
$$

Note that the adjustment factors $C_F = 1$ is temporary. Once an initial beam size is found, the section modulus will be re-calculated.

$$
S_{min} = 49.33 \,\mathrm{in}^3 \tag{77}
$$

This Section modulus allows for lumber of sizes 3x12 and 4x10. Adjusting the size factor to 1.15, a size of 2x14 is also allowable.

The cable will be mounted to the point on the roof perpendicular to its mounting point on the beam at line A. A roof slope of 4:12 is expected. Using similar triangles and the given dimensions of the tiny house, the vertical spanning distance of the cable is found to be 3.375'. The cable length is found to be 7.77'.

Using $\frac{L}{2}$ $_y$ = 2214.84 lb, as found earlier, the force of the cable is found to be $F_C = 5099.9$ lb. From data on McMaster-Carr, a 1/2" steel wire rope, which has a capacity for 5300 lb, is deemed appropriate for this design.

For the post design, the shear force at the end of the beam, $F = 664.46$ lb, is used.

$$
\frac{F}{A} = \sigma = F_C * C_P * C_D * C_F = 475 * C_P * 1.15 * 1 \tag{78}
$$

$$
F_C* = 546.25\tag{79}
$$

Using equation 44, and assuming $2x$, C_P is found to equal 0.1634

$$
\frac{664.46}{A} \le 475 * 0.1634 * 1.15 * 1 \to A = 7.44 \,\text{in}^2 \tag{80}
$$

A 2x6 post satisfies this area constraint.

5.8 Multiple Post Design

While some of the above designs can be used, all require very large beams and posts. The group concluded that such large members would detract from the aesthetics of the trellis, so options with more supports have been explored. With the supports evenly spaced along the beam, SAP2000 was used to rapidly find which configuration would allow for the most reasonably sized members. This yielded a design with 4 supports evenly spaced along the length of the 27' beam.

Figure 15: 4 Post Design

The beam is indeterminate, so the Force Method is employed to find reactions at the supports.

The statically determinate beam is a pin-roller beam with supports at locations A and D, as seen in Figure 15.

$$
\Delta_B = \frac{w(9')}{24EI} (27'^3 - 2(27')(9')^2 + (9')^3 = 7.894 \times 10^5 / EI \tag{81}
$$

Assume $F_B = F_C$. The redundant at B applies a deflection of:

$$
\Delta_{RB} = \frac{F_B(9')}{6EI} (3(9')(27') - 3(9')^2 - (9')^2 = 6.075 \times 10^2 F_B/EI
$$
 (82)

The compatibility equation is $\Delta_B + \Delta_{RB} = 0$. From this, we find

$$
F_B = F_C = 1299 \,\text{lb} \tag{83}
$$

$$
\sum M_A = 9'(F_B) + 18'(F_C) + 27'(F_D) - 13.5'(131.25 \times 27') = 0 \rightarrow F_D = 472.9 \,\text{lb}
$$
\n(84)

$$
\sum F_y = -131.25 \times 27' + F_A + F_B + F_C + F_D = 0 \rightarrow F_A = 472.9 \,\text{lb} \tag{85}
$$

$$
M_{max} = M_B = 4.5'(131.25(9') - 9'(F_A) = 1059.5 \,\text{lb} * \text{ft} \tag{86}
$$

$$
F'_B = 727.375 \,\text{psi} > \frac{1059.5 \times 12}{S_{min}} \to S_{min} = 13.5 \,\text{in}^3 \tag{87}
$$

The two options that satisfy the minimum section modulus are 2x10 and 4x6 lumber. 2x10 is less expensive, so it is the chosen lumber for the design.

For column sizing, $C_D = 1.15$ and $C_F = 1.15$. Using Equation 44, $C_P =$ 0.305.

$$
F_C' = 475 \text{psi} * 1.15 * 1.15 * 0.305 = 191.54 \tag{88}
$$

$$
\frac{1300 \text{lb}}{A} < 191.54 \text{psi} \to A > 6.79 \text{ in}^2 \tag{89}
$$

The appropriate column sizing is therefore 3x4.

5.9 Multiple Cables and Posts Design

Figure 16: Design for cables and posts, spacing 9' between supports

Another option is to use four supports, as shown in the multiple post design, but to convert the rollers at B and C to cables. These cables would undergo the same vertical force as calculated for the posts. The cables will be connected to the tallest point on the roof.

Figure 17: $F_y = 1299$ lb, $F_{xy} =$ √ $2F_y = 1837$ lb

Figure 18: $F_{xy} = 1837$ lb, $F_{xyz} = \frac{\sqrt{7^2+9^2}}{9} F_{xy} = 2327$ lb

For cable B, $F_y = 1299$ lb, and given the known dimensions, the force can be found as $F = 2327$ lb. This requires a 3/8" wire rope rated at 3,000 lb according to McMaster-Carr.

Figure 19: $F_y = 1299$ lb, $F_{xy} = \frac{\sqrt{9^2 + 18^2}}{9} F_y = 2905$ lb

Figure 20: $F_{xy} = 2905$ lb, $F_{xyz} = \frac{\sqrt{7^2 + 9^2}}{9} F_{xy} = 3680$ lb

For cable C, with the same vertical force, the cable force can be found as $F = 3680$ lb. This requires a 7/16" wire rope rated at 4,000 lb according to McMaster-Carr.

6 Model and Drawings for Final Design

The final recommended design is the multiple post design. This option is most cost effective and allows for the most reasonably sized members.

An alternative to this design is the multiple post and cable design. The two posts in the center of the beam could be replaced with two cables. We did not choose this model as our final design because of its high cost.

Figure 21: Drawing for All Post Design

Figure 22: Built Trellis Model

7 Wood Properties

Figure 23: Properties of Northern White Cedar